

Wrapping corrections beyond the $\mathfrak{sl}(2)$ sector in $\mathcal{N} = 4$ SYM

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The $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM theory has been much studied and the anomalous dimensions of those operators are well known. Nevertheless, many interesting operators are not included in this sector. We consider a class of twist operators beyond the $\mathfrak{sl}(2)$ subsector introduced by Freyhult, Rej and Zieme. They are spin n , length-3 operators. At one-loop they can be identified with three gluon operators. At strong coupling, they are associated with spinning strings with two spins in AdS space and charge in S^5 . We exploit the Y-system to compute the leading weak-coupling four loop wrapping correction to their anomalous dimension. The result is written in closed form as a function of the spin n . We combine the wrapping correction with the known four loop asymptotic Bethe Ansatz contribution and analyze special limits in the spin n . In particular, at large n , we prove that a generalized Gribov-Lipatov reciprocity holds. At negative unphysical spin, we present a simple BFKL-like equation predicting the rightmost leading poles.

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1 Introduction

The mirror thermodynamic Bethe Ansatz (TBA) for the $AdS_5 \times S^5$ superstring [1] provides a general approach to the study of finite size corrections to states/operators in AdS/CFT. The associated Y-system has been proposed in [2] based on symmetry arguments and educated guesses about the analyticity and asymptotic properties of the Y-functions.

Very powerful explicit tests of these methods have been mostly done in the $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM [3]. Beyond $\mathfrak{sl}(2)$, our knowledge of the larger part of the full $\mathfrak{psu}(2, 2|4)$ structure of the theory is limited.

In this brief note, following [4], [5], we show some recent progress in the computation of the leading order wrapping corrections of the Freyhult-Rej-Zieme (FRZ) operators, a special class of operators beyond the $\mathfrak{sl}(2)$ sector first discussed at one-loop in the general analysis [6] and further analyzed at all orders in [7]. The results follow from a direct application of the Y-system techniques.

FRZ operators are twist 3 operators. Their general form can be written by inserting covariant and anti-covariant derivatives \mathcal{D} , $\bar{\mathcal{D}}$ into the half-BPS state $\text{Tr} \mathcal{Z}^3$ (\mathcal{Z} being one of the three complex scalars of $\mathcal{N} = 4$ SYM theory):

$$\mathbb{O}_{n,m}^{\text{FRZ}} = \text{Tr} (\mathcal{D}^{n+m} \bar{\mathcal{D}}^m \mathcal{Z}^3) + \dots \quad (1)$$

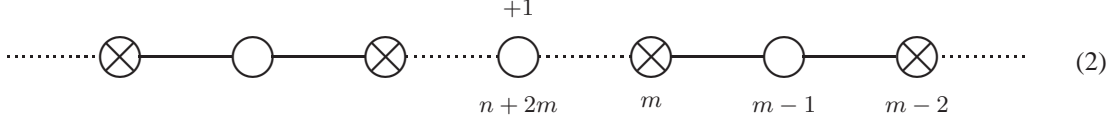
At strong coupling, these operators are dual to spinning string configurations with two spins $S_1 = n+m-\frac{1}{2}$ and $S_2 = m - \frac{1}{2}$ in AdS_5 and charge $J = L = 3$ in S^5 . The distribution of the roots on the nodes of the

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$\mathfrak{psu}(2, 2|4)$ algebra is summarized in Figure 2:



Non trivial roots out of the $\mathfrak{sl}(2)$ sector occur for $m \geq 2$. In the particular $m = 2$ case, the operators are called 3-gluon operators.

In general, the wrapping effects are expected to appear at weak coupling for the $\mathbb{O}_{n,m}^{\text{FRZ}}$ operators at four loop. In paper [8], the asymptotic part of the anomalous dimensions of the $\mathbb{O}_{n,2}^{\text{FRZ}}$ has been computed exactly up to four loops. The result is given as a closed formula in the spin parameter n . Up to this level, reciprocity holds for the asymptotic contributions. By following [4], from the Y-system equations we derive precisely a similar closed formula for the leading wrapping corrections. This formula, together with the results of [8], completes the study of the anomalous dimensions for the 3-gluon operators up four loop. As a byproduct, we shall be able to test positively reciprocity as well as discuss the BFKL poles of the full four loop result. We also show that a very simple and natural modification of the twist-2 BFKL equation predicts the correct pole structure.

For the general $\mathbb{O}_{n,m}^{\text{FRZ}}$ FRZ operators, no asymptotic closed formula for the anomalous dimension is known beyond one-loop. Thus, the research of such a closed formula for the wrapping corrections is hopeless and useless. Nevertheless the large n expansion of the asymptotic minimal anomalous dimension of $\mathbb{O}_{n,m}^{\text{FRZ}}$ is known for fixed ratio n/m or fixed m [7]. The expansion is obtained at all orders in the coupling and including the leading term $\sim \log n$ as well as the subleading asymptotically constant correction $\sim n^0$. These two contributions are expected to be free of wrapping corrections.

Following [5], we consider precisely the large n expansion of leading order wrapping correction which appears at four loop. We provide an algorithm to compute through the Y-system the large n expansion for fixed m and present explicit new results for $m = 2, 3, 4$. For $m = 2$ we match the large n expansion of the closed formula for the 3-gluon operators [4]. The expansions for the other two values are new. In full generality, we prove the $\frac{\log n}{n^2}$ scaling behaviour at large n thus confirming the assumption in [7].

The plan of the paper is the following: In the next Section we give the minimal set of Y-system equations required to compute the leading order wrapping corrections. In Section 3 we give the result for the leading wrapping corrections of the 3-gluon operators. In Section 4 we consider the large n limit of FRZ operators $\mathbb{O}_{n,m}^{\text{FRZ}}$. The general strategy is described and results are given for $m = (3, 4)$.

2 Leading wrapping corrections from Y-system

The Y-system is a set of functional equations for the functions $Y_{a,s}(u)$ defined on the fat-hook of $\mathfrak{psu}(2, 2|4)$ [2]. These equations are

$$\frac{Y_{a,s}^+ Y_{a,s}^-}{Y_{a+1,s} Y_{a-1,s}} = \frac{(1 + Y_{a,s+1})(1 + Y_{a,s-1})}{(1 + Y_{a+1,s})(1 + Y_{a-1,s})}. \quad (3)$$

Their boundary conditions are discussed in [9]. The anomalous dimension of a generic state is given by the TBA formula

$$E = \sum_{\ell=0}^{\infty} g^{2\ell} \gamma_{\ell\text{-loop}} = \underbrace{\sum_i \epsilon_1(u_{4,i})}_{\text{asymptotic } \gamma^{\text{asy}}} + \underbrace{\sum_{a \geq 1} \int_{\mathbb{R}} \frac{du}{2\pi i} \frac{\partial \epsilon_a^*}{\partial u} \log(1 + Y_{a,0}^*(u))}_{\text{wrapping W}}, \quad (4)$$

where the asymptotic and the wrapping contributions are well distinct. In formula (4), the dispersion relation is

$$\epsilon_a(u) = a + \frac{2ig}{x^{[a]}} - \frac{2ig}{x^{[-a]}}, \quad (5)$$

and the star means evaluation in the mirror kinematics¹.

The crucial assumption in the identification of relevant solutions to the Y-system is

$$Y_{a \geq 1, 0}^* \sim \left(\frac{x^{[-a]}}{x^{[+a]}} \right)^L, \quad (6)$$

for large L , or large u (or small g). In this limit, it can be shown that the Hirota equation splits in two $\mathfrak{su}(2|2)_{L,R}$ wings. One can have a simultaneous finite large L limit on both wings after a suitable gauge transformation. The solution is then

$$Y_{a,0}(u) \simeq \left(\frac{x^{[-a]}}{x^{[+a]}} \right)^L \frac{\phi^{[-a]}}{\phi^{[+a]}} T_{a,1}^L T_{a,1}^R, \quad (7)$$

where $\frac{\phi^{[-a]}}{\phi^{[+a]}}$ is the fusion form factor and $T_{a,1}^{L,R}$ are the transfer matrices of the antisymmetric rectangular representations of $\mathfrak{su}(2|2)_{L,R}$. They can be explicitly computed by an appropriate generating functional [10].

At weak coupling, the formula (4) for the leading wrapping corrections takes the simpler form

$$W = -\frac{1}{\pi} \sum_{a=1}^{\infty} \int_{\mathbb{R}} du Y_{a,0}^* = -2i \sum_{a=1}^{\infty} \text{Res}_{u=\frac{ia}{2}} Y_{a,0}^*. \quad (8)$$

The formula (7) for the relevant Y functions gets simplified too. In particular, the Y 's can be expressed as functions of the 1-loop Baxter polynomials Q_i 's. We get [5]:

$$\begin{aligned} \left(\left(\frac{x^{[-a]}}{x^{[+a]}} \right)^L \right)^* &= \left(\frac{4g^2}{a^2 + 4u^2} \right)^L, \\ \left(\frac{\phi^{[-a]}}{\phi^{[+a]}} \right)^* &= [Q_4^+(0)]^2 \frac{Q_4^{[1-a]}}{Q_4^{[-1-a]} Q_4^{[a-1]} Q_4^{[a+1]}} \frac{Q_5^{[-a]}}{Q_5(0)} \frac{Q_7(0)}{Q_7^{[-a]}}, \\ T_{a,1}^{*,L} &= (\log(Q_4')) \Big|_{u=-\frac{i}{2}}^{u=+\frac{i}{2}} g^2 \frac{(-1)^{a+1}}{Q_4^{[1-a]}} \sum_{\substack{k=-a \\ \Delta k=2}}^a \frac{Q_4^{[-1-k]} - Q_4^{[1-k]}}{u - i\frac{k}{2}} \Big|_{Q_4^{[-1-a]}, Q_4^{[-1-a]} \rightarrow 0} + \mathcal{O}(g^4), \\ T_{a,1}^* &= (-1)^{a+1} \frac{Q_5^{[a]} Q_7^{[-a]}}{Q_4^{[1-a]}} \sum_{\substack{k=1-a \\ \Delta k=2}}^{a-1} \frac{Q_6^{[k]}}{Q_6^{[k]}} \left(\frac{Q_6^{[k+2]} - Q_6^{[k]}}{Q_5^{[k+1]} Q_7^{[k+1]}} + \frac{Q_6^{[k-2]} - Q_6^{[k]}}{Q_5^{[k-1]} Q_7^{[k-1]}} \right) + \mathcal{O}(g^2). \end{aligned} \quad (9)$$

The expressions for the Q_i 's for the FRZ operators are known and can be found in [7], [8]. These expressions together with equations (7), (8) and (9) are in principle all what one needs to compute the leading wrapping corrections for the FRZ operators.

3 The 3-gluon operators $\mathbb{O}_{n,2}^{\text{FRZ}}$ case

FRZ operators $\mathbb{O}_{n,2}^{\text{FRZ}}$ with $m = 2$ are among the simplest non trivial operators that can be built beyond the $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM theory. In this Section we give a general result for their leading order wrapping corrections W_n . More details about the results of this section can be found in [4].

The starting point is the observation that by specializing the formulas of Section 2 to the simple $m = 2$ case and fixing n , the computation of the wrapping corrections is straightforward. In [5] we produced a list

¹ Shifted quantities are defined as $F^{\overbrace{\pm \dots \pm}_a}(u) = F^{[\pm a]}(u) = F(u \pm i\frac{a}{2})$.

of results for the wrapping up to $n = 70$. Then we were able to condensate these data in a closed formula that replicates and generalizes to any value of n the numerical results of the list. The formula is

$$\begin{aligned}
W_n &= (r_{0,n} + r_{3,n} \zeta_3 + r_{5,n} \zeta_5) g^8, \\
r_{5,n} &= 80 \left(4S_1 + \frac{2}{N+1} + 4 \right) \left(-4(N+1) + \frac{1}{N+1} \right), \\
r_{3,n} &= 16 \left(4S_1 + \frac{2}{N+1} + 4 \right) \times \\
&\quad \left[8(N+1)S_2 + 8 + \frac{2}{N+1}(2-S_2) - \frac{2}{(N+1)^2} - \frac{1}{(N+1)^3} \right], \\
r_{0,n} &= 2 \left(4S_1 + \frac{2}{N+1} + 4 \right) \times \\
&\quad \left[16(N+1)(2S_{2,3} - S_5) + 32S_3 + \frac{4}{N+1}(S_5 - 2S_{2,3} + 4S_3) + \right. \\
&\quad \left. + \frac{8}{(N+1)^2}(-S_3 + 2) + \frac{4}{(N+1)^3}(-S_3 + 4) - \frac{4}{(N+1)^5} - \frac{1}{(N+1)^6} \right]. \quad (10)
\end{aligned}$$

Here $S_{a,b,\dots} \equiv S_{a,b,\dots}(N)$ and $N = n/2 + 1^2$. Note that each of the rational coefficient r_i can be written as $r_{i,n} = \gamma_{1\text{-loop}} \tilde{r}_{i,n}$, where $\gamma_{1\text{-loop}} = S_1 + \frac{2}{N+1} + 4$ is the one loop anomalous dimension.

The Ansatz (10) completes the four loop expression of the energy spectrum for the $\mathbb{O}_{n,2}^{\text{FRZ}}$ operators, the other relevant contributions up to this order being the first four asymptotic orders. Their expressions can be found in [8].

Up to four loop, the asymptotic part of the spectrum shows the generalized Gribov-Lipatov reciprocity property [8]. Formula (10) allows to check whether this property extends to the full four loop result. This is indeed the case: The large n expansion of $W_n/\gamma_{1\text{-loop}}$ reads

$$\begin{aligned}
\zeta_5 \tilde{r}_{5,n} + \zeta_3 \tilde{r}_{3,n} + \tilde{r}_{0,n} = & \frac{\frac{32}{3} - 32\zeta_3}{J^2} + \frac{\frac{232\zeta_3}{5} - \frac{352}{15}}{J^4} + \frac{\frac{4834}{105} - \frac{2344\zeta_3}{35}}{J^6} + \frac{\frac{3544\zeta_3}{35} - \frac{83956}{945}}{J^8} + \frac{\frac{271768}{1485} - \frac{9512\zeta_3}{55}}{J^{10}} + \\
& \frac{\frac{1872392\zeta_3}{5005} - \frac{20053258}{45045}}{J^{12}} + \frac{\frac{87933002}{61425} - \frac{524872\zeta_3}{455}}{J^{14}} + \frac{\frac{4917304\zeta_3}{935} - \frac{5747755528}{883575}}{J^{16}} + \dots, \quad (11)
\end{aligned}$$

where we introduced the charge $J^2 = N(N+2)$. All the odd powers of $1/J$ cancel proving that the reciprocity property does hold. It is remarkable that this property is a consequence of non-trivial cancellations of odd $1/J$ terms which are present in the expansion of each single coefficient $\tilde{r}_{i,n}$. The presence of reciprocity is really appreciable, since it allows to predict a half of the large n expansion terms (expressed as functions of the same n) as combinations of the other half.

The Ansatz (10) allows also to study the BFKL poles of the full four loop result. In general, at ℓ -loop the analytic continuation of $\gamma_{\ell\text{-loop}}$ in the variable N around $N = -1$ is expected to behave at worst as $\omega^{-\ell}$, where ω is a small expansion parameter defined by $N = -1 + \omega$. At four loops the asymptotic anomalous dimension $\gamma_{4\text{-loop}}^{\text{asy}}$ presents instead also poles in ω^{-k} with $k = (7, 6, 5)$. These poles get indeed compensated inside the full $\gamma_{4\text{-loop}} = \gamma_{4\text{-loop}}^{\text{asy}} + W$ where precisely the wrapping contribution (10) is included. The final expressions for the expansions of the first four $\gamma_{\ell\text{-loop}}$ are

$$\begin{aligned}
\gamma_{1\text{-loop}} &= -\frac{4}{\omega} + \dots, & \gamma_{2\text{-loop}} &= \frac{8}{\omega^2} + \frac{4\pi^2}{3\omega} + \dots, \\
\gamma_{3\text{-loop}} &= \frac{0}{\omega^3} - \frac{16(-3\zeta_3 + \pi^2 + 12)}{3\omega^2} + \dots, & \gamma_{4\text{-loop}} &= -\frac{32(1 + 2\zeta_3)}{\omega^4} + \frac{160\zeta_3}{\omega^3} + \dots
\end{aligned} \quad (12)$$

² We remind that n is an even positive integer. This means that N is an integer, $N \geq 2$.

Strikingly, the leading poles can be reproduced by a BFKL-like equation that links ω to the full anomalous dimension γ

$$-\frac{\omega}{g^2} = \chi_1\left(\frac{\gamma}{2}\right), \quad \chi_1(z) = S_1(z) + S_1(z+1). \quad (13)$$

In fact, the weak coupling expansion of this equation reads precisely

$$\gamma = \left(-\frac{4}{\omega} + \dots\right) g^2 + \left(\frac{8}{\omega^2} + \dots\right) g^4 + \left(\frac{0}{\omega^3} + \dots\right) g^6 + \left(-\frac{32(1+2\zeta_3)}{\omega^4} + \dots\right) g^8 + \dots \quad (14)$$

4 General FRZ operators: Large n expansion

For the general $\mathbb{O}_{n,m>2}^{\text{FRZ}}$ only the large n limit of the asymptotic part of the anomalous dimension is known for fixed ratio n/m or fixed m [7]. Thus, as far as we consider the wrapping corrections, we are primarily interested in the computation of their large n expansions.

The Y-system described in Sec. 2 can be optimized to get only the large n contributions. The computational strategy is the following:

- Starting from eq. (8) one evaluates the residue at fixed $a = 1, 2, \dots$ without assigning n ;
- Analyzing the dependence on n of the residue, one realizes that n comes from the Baxter polynomials Q_4 , its derivatives (which are written in terms of the basic hypergeometric function $F_{n,m}$) and from the explicit n -dependent coefficients of the other Baxter polynomials. At this point the limit over n can be taken in two distinct steps;
- Using the Baxter equation, it is possible to shift the argument of $F_{n,m}$ to some minimal value and take the large n limit on the coefficients. This gives a first expansion containing various derivatives of the logarithm of $F_{n,m}$;
- The derivatives of the logarithm can be systematically computed by means of the method explained in [11];
- The outcome of this procedure are sequences of rational numbers being the a -dependent coefficients of the large n expansion of $\text{Res}_{u=i\frac{a}{2}} Y_{a,0}^*$. These sequences turn out to be rather simple rational functions which are easily identified. The sum over a of these rational functions gives the wrapping W as defined by eq. (8).

Following this computational method, in [5] we computed the large n expansion of the wrapping corrections for the operators $\mathbb{O}_{n,m}^{\text{FRZ}}$ with $m = 2, 3, 4$. The $m = 2$ case agrees with the large n expansion of eq. (10). For $m = 3, 4$ we get these new results

$$g^{-8} W_{n,m=3} = -\frac{4}{3} (36\zeta_3 + 5\pi^2 - 3) \frac{2 \log \bar{n} + 1}{n^2} + \frac{88}{3} (36\zeta_3 + 5\pi^2 - 3) \frac{\log \bar{n}}{n^3} + \dots \quad (15)$$

$$-\frac{2}{9n^4} [4(9108\zeta_3 + 1289\pi^2 - 615) \log \bar{n} - 14868\zeta_3 - 2017\pi^2 + 1527] + \dots,$$

$$g^{-8} W_{n,m=4} = -\frac{1024}{1215} (81\zeta_3 - 32) \frac{3 \log \frac{\bar{n}}{2} + 4}{n^2} + \frac{7168}{1215} (81\zeta_3 - 32) \frac{6 \log \frac{\bar{n}}{2} + 5}{n^3} + \dots \quad (16)$$

$$-\frac{1024}{8505n^4} [138240(1971\zeta_3 - 760) \log \frac{\bar{n}}{2} + 143289\zeta_3 - 53248] + \dots$$

The extension of these results to larger values of m is a plain task.

The numerical check in Table (17) shows that there is good agreement between our expansions and a numerical estimate of the wrapping corrections.

$m = 3$	n	estimate	(LO	NLO	NNLO)	full expansion	diff %
	10	-1.857411	(-4.038730	3.785374	-2.548066)	-2.801422	51%
	30	-0.438781	(-0.594614	0.193683	-0.045355)	-0.446286	1.7%
	50	-0.197270	(-0.238478	0.047207	-0.006716)	-0.197986	0.36%
$m = 4$	n	estimate	(LO	NLO	NNLO)	full expansion	diff %
	10	-1.201460	(-2.908787	3.493848	-3.576129)	-2.991068	149%
	30	-0.293765	(-0.424071	0.176476	-0.061888)	-0.309484	5.4%
	50	-0.134246	(-0.169551	0.042847	-0.009090)	-0.135794	1.2%

(17)

Following the general idea of reciprocity, we are led to rewrite the above large n expansions in terms of the quantity

$$J_m^2 = n(n + a_m). \quad (18)$$

It turns out that the coefficients of the two odd terms $1/J^3$ and $\log J/J^3$ indeed vanish for the choice $a_{2,3,4} = 8, 11, 14$. This is not completely trivial since we have one parameter and two structures. It is tempting to conjecture the simple relation $a_m = 3m + 2$ and to claim that reciprocity in the above sense holds for the full anomalous dimension as well.

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